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Confinement effects from interacting chromo-magnetic and axion fields

Patricio Gaete[1] **and Euro Spallucci**[2]

[1] Departamento de Física, Universidad Técnica F. Santa María, Valparaíso, Chile
[2] Dipartimento di Fisica Teorica, Università di Trieste and INFN, Sezione di Trieste, Italy

E-mail: patricio.gaete@usm.cl and spallucci@ts.infn.it

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Abstract

We study a non-Abelian gauge theory with a pseudo scalar coupling $\phi\,\mathrm{Tr}(F^{*}_{\mu\nu}F^{\mu\nu})$ in the case where a constant chromo-electric, or chromo-magnetic, strength expectation value is present. We compute the interaction potential within the framework of gauge-invariant, path-dependent, variables formalism. While in the case of a constant chromo-electric field strength expectation value the static potential remains Coulombic, in the case of a constant chromo-magnetic field strength the potential energy is the sum of a Coulombic and a linear potential, leading to the *confinement* of static charges.

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1. Introduction

It is presently widely accepted that quantitative understanding of confinement of quarks and gluons remains as the major challenge in *QCD*. In this connection, a linearly increasing quark–antiquark pair static potential provides the simplest criterion for confinement, although unfortunately there is up to now no known way to analytically derive the confining potential from first principles. In this context, it may be recalled that phenomenological models have been of considerable importance in order to provide strong insight into the physics of confinement, and can be considered as effective theories of *QCD*. One of these, which is the dual superconductivity picture of *QCD* vacuum [1], has probably enjoyed the greatest popularity. The key ingredient in this model is the condensation of topological defects originated from quantum fluctuations (monopoles). As a consequence, the colour electric flux linking quarks is squeezed into 'strings', and the nonvanishing string tension represents the proportionality constant in the linear, quark confining, potential. We further note that recently an interesting approach to this problem has been proposed [2], which includes the contribution of all topologically nontrivial sectors of a gauge theory. Mention should be made, at this point, to lattice calculations which clearly show the formation of tubes of gluonic fields connecting coloured charges [3]. In agreement with lattice results, loop–loop correlations have been recently computed analytically in extended stochastic vacuum models [4].

With these ideas in mind, in a previous paper [5], we have studied a simple effective theory where confining potentials are obtained in the presence of nontrivial constant expectation values for the gauge field strength $F_{\mu\nu}$ coupled to a scalar (axion) field ϕ, via the interaction term

$$\mathcal{L}_I = \frac{g}{8}\phi\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}. \tag{1}$$

In particular, we have observed that in the case of a constant electric field strength expectation value the static potential remains Coulombic, while in the case of a constant magnetic field strength expectation value the potential energy is the sum of a Yukawa and a linear potential, leading to the confinement of static charges. More interestingly, we remark that the magnetic character of the field strength expectation value needed to obtain confinement is in agreement with the current chromo-magnetic picture of the *QCD* vacuum [6]. Another feature of this model is that it restores the rotational symmetry (in the potential), despite that the external fields break this symmetry. We further observe that similar results have been obtained in the context of the dual Ginzburg–Landau theory [7], as well as, for a theory of antisymmetric tensor fields that results from the condensation of topological defects as a consequence of the Julia–Toulouse mechanism [8]. Accordingly, from a phenomenological point of view, we have established an equivalence between different models describing the same physical phenomena. This allows us to obtain more information about a theory than is possible by considering a single description.

By following this line of reasoning, it is natural to extend the previous analysis to the case where a scalar field ϕ is coupled to a *non-Abelian* gauge field, that is,

$$\mathcal{L}_I = \frac{\beta}{8}\phi\,\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,\mathbf{F}_{\mu\nu}\,\mathbf{F}_{\alpha\beta}, \tag{2}$$

where the trace is taken over colour indices. While an Abelian model with interaction of the type (1) is no more than a useful ‘laboratory’, it may be worth to recall that in *QCD* coupling of the form (2) is instrumental to build up the Peccei–Quinn mechanism [12] and solve the strong CP problem [9–11]. In this case, the scalar field describes the axion, i.e. the Nambu–Goldstone boson of a new broken $U(1)$ symmetry of the quark and Higgs sector.

The purpose of this work is to extend the Abelian calculations in [5] to the non-Abelian case and find the corresponding static potential. Our calculations are done within the framework of the gauge invariant/path-dependent variables formalism, providing an effective tool for a better understanding of effective non-Abelian theories. One important advantage of this approach is that it provides a physically-based alternative to the usual Wilson loop approach, where in the latter the usual qualitative picture of confinement in terms of an electric flux tube linking quarks emerges naturally. As we shall see, in the case of a constant chromo-electric field strength expectation value the static potential remains Coulombic. On the other hand, in the case of a constant chromo-magnetic field strength expectation value the potential energy is the sum of a Coulombic and a linear potential, that is, the confinement between static charges is obtained. As a result, the new coupling displays a marked departure of a qualitative nature from the results of [5] at large distances.

It is interesting to observe that the dual superconductivity picture of *QCD*, compared with the model proposed here, involves the condensation of topological defects originated from quantum fluctuations. Thus one is led to the conclusion that the phenomenological model proposed here incorporates automatically the contribution of the condensate of topological defects to the vacuum of the model or, alternatively, the nontrivial topological sectors as in [2]. Another way of obtaining the above conclusion is by invoking the bosonization technique in (1 + 1) dimensions. Indeed, it is a well-known fact that, for instance, in the Schwinger model [13], the bosonized version (effective theory) contains quantum corrections at the classical level. In the same way, we can interpret the model proposed here as an effective theory which contains quantum effects at the classical level. Thus, one obtains a similarity between the tree level mechanism that leads to confinement here and the nonperturbative mechanism which gives confinement in *QCD*. Accordingly, the above interrelations are interesting from the point of view of providing unifications among diverse models as well as exploiting the equivalence in explicit calculations, as we are going to show.

2. Interaction energy

As we discussed in the introduction, our immediate objective is to calculate explicitly the interaction energy between static pointlike sources, for a model containing the term (2), along the lines of [5, 15]. To this end we will compute the expectation value $\langle H\rangle_\Phi$ of the Hamiltonian operator H in the physical state $|\Phi\rangle$ describing the sources. The non-Abelian gauge theory we are considering is defined by the following generating functional in four-dimensional spacetime:

$$\mathcal{Z} = \int \mathcal{D}\phi\mathcal{D}A \exp\left\{-\mathrm{i}\int \mathrm{d}^4x\mathcal{L}\right\}, \tag{3}$$

with

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\,\mathbf{F}_{\mu\nu}\,\mathbf{F}^{\mu\nu} + \frac{\beta}{8}\,\phi\,\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\,\mathbf{F}_{\mu\nu}\,\mathbf{F}_{\rho\sigma} + \frac{1}{2}\partial_\mu\,\phi\,\partial^\mu\,\phi + \frac{m_A^2}{2}\phi^2, \tag{4}$$

where m_A is the mass for the axion field ϕ. Here, $\mathbf{A}_\mu(x) = A_\mu^a(x)\,T_a$, where T_a is a Hermitian representation of the semi-simple and compact gauge group, and $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{bc}^a A_\mu^b A_\nu^c$, with f_{bc}^a the structure constants of the group. As in [5] we restrict ourselves to static scalar fields; a consequence of this is that one may replace $\partial^2\phi = -\nabla^2\phi$. It also implies that, after performing the integration over ϕ in $\mathcal{Z}$, the effective Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\,\mathbf{F}_{\mu\nu}\,\mathbf{F}^{\mu\nu} + \frac{\beta^2}{128}\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\,\mathbf{F}_{\mu\nu}\,\mathbf{F}_{\rho\sigma}\frac{1}{\nabla^2 - m_A^2}\varepsilon^{\alpha\beta\gamma\delta}\mathrm{Tr}\,\mathbf{F}_{\alpha\beta}\,\mathbf{F}_{\gamma\delta}. \tag{5}$$

Furthermore, as was explained in [14], expression (5) can be rewritten as

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\,\mathbf{f}_{\mu\nu}\,\mathbf{f}^{\mu\nu} + \frac{\beta^2}{16}\varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\langle\mathbf{F}_{\mu\nu}\rangle\,\mathbf{f}_{\alpha\beta}\frac{1}{\nabla^2 - m_A^2}\varepsilon^{\rho\sigma\gamma\delta}\mathrm{Tr}\,\langle\mathbf{F}_{\rho\sigma}\rangle\,\mathbf{f}_{\gamma\delta}. \tag{6}$$

where $\langle F_{\mu\nu}^a\rangle$ represents the constant classical background (which is a solution of the classical equations of motion). Here, $f_{\mu\nu}^a$ describes a small fluctuation around the background; we also mention that the above Lagrangian arose after using $\varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\,\langle\mathbf{F}_{\mu\nu}\rangle\,\langle\mathbf{F}_{\alpha\beta}\rangle = 0$ (which holds for a pure chromo-electric or a pure chromo-magnetic background).

By introducing the notation $\varepsilon^{\mu\nu\alpha\beta}\langle\mathbf{F}_{\mu\nu}\rangle \equiv \mathbf{v}^{\alpha\beta}$ and $\varepsilon^{\rho\sigma\gamma\delta}\langle\mathbf{F}_{\rho\sigma}\rangle \equiv \mathbf{v}^{\gamma\delta}$, expression (6) then becomes

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\,\mathbf{f}_{\mu\nu}\mathbf{f}^{\mu\nu} + \frac{\beta^2}{16}\mathrm{Tr}\,\mathbf{v}^{\alpha\beta}\mathbf{f}_{\alpha\beta}\frac{1}{\nabla^2 - m_A^2}\mathrm{Tr}\,\mathbf{v}^{\gamma\delta}\mathbf{f}_{\gamma\delta}. \tag{7}$$

At this stage we note that (7) has the same form as the corresponding Abelian effective Lagrangian density. This common feature is our main motivation to study the effect of the non-Abelian coupling on the interaction energy.

2.1. Chromo-magnetic case

We now proceed to obtain the interaction energy in the $\mathbf{v}^{oi} \neq 0$ and $\mathbf{v}^{ij} = 0$ case (referred to as the chromo-magnetic one in what follows), by computing the expectation value of the Hamiltonian in the physical state $|\Phi\rangle$. The Lagrangian (7) then becomes

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\,\mathbf{f}_{\mu\nu}\mathbf{f}^{\mu\nu} + \frac{\beta^2}{4}\mathrm{Tr}\,\mathbf{v}^{0i}\mathbf{f}_{0i}\frac{1}{\nabla^2 - m_A^2}\mathrm{Tr}\,\mathbf{v}^{0k}\mathbf{f}_{0k} - \mathrm{Tr}\,\mathbf{A}_0\mathbf{J}^0, \tag{8}$$

where $\mathbf{J}^0$ is an external current, $\mu, \nu = 0, 1, 2, 3$ and $i, k = 1, 2, 3$. Once this is done, the canonical quantization in the manner of *Dirac* yields the following results. The canonical momenta are

$$\Pi^{a0} = 0, \tag{9}$$

$$\Pi_i^a = D_{ij}^{ab} E_b^j, \tag{10}$$

$$E_i^a \equiv f_{i0}^a, \tag{11}$$

$$D_{ij}^{ab} \equiv \left(\delta^{ab}\delta_{ij} + \frac{\beta^2}{2} v_{i0}^a \frac{1}{\nabla^2 - m_A^2} v_{j0}^b \right). \tag{12}$$

Since $\mathbf{D}$ is nonsingular, there exists the inverse $\mathbf{D}^{-1}$ and from equation (10) we obtain

$$E_i^a = \frac{1}{\det \mathbf{D}} \left\{ \delta^{ab}\delta_i^j \det \mathbf{D} - \frac{\beta^2}{2} v_{i0}^a \frac{1}{\nabla^2 - m_A^2} v_{j0}^b \right\} \Pi^{bj}. \tag{13}$$

The corresponding canonical Hamiltonian is thus

$$H_C = \int \mathrm{d}^3x\, \mathrm{Tr} \left[\mathbf{\Pi}^i (\mathbf{D}\mathbf{A}_0)_i + \frac{1}{2}\mathbf{\Pi}^i\mathbf{\Pi}_i + \frac{1}{2}\mathbf{B}^i\mathbf{B}_i - \frac{\beta^2}{4}(\mathbf{v}^i\mathbf{\Pi}_i)\frac{1}{\nabla^2 - M^2}(\mathbf{v}^i\mathbf{\Pi}_i) + (\mathbf{A}_0\mathbf{J}^0) \right], \tag{14}$$

where $M^2 \equiv m_A^2 - \frac{\beta^2}{8}\mathbf{v}^i\mathbf{v}_i$ and $\mathbf{B}^i$ is the *chromo-magnetic* field. By applying the Dirac quantization procedure for constrained systems, and removing non-physical variables by imposing an appropriate gauge condition[3], we can compute the interaction energy between pointlike sources in the model under consideration. A fermion is localized at $\mathbf{0}$ and an antifermion at $\mathbf{y}$. From our above discussion, we see that $\langle H \rangle_\Phi$ reads

$$\langle H \rangle_\Phi = \langle \Phi | \int \mathrm{d}^3x\, \mathrm{Tr} \left[\frac{1}{2}\mathrm{Tr}\,\mathbf{\Pi}^i\mathbf{\Pi}_i - \frac{\beta^2}{4}\mathrm{Tr}\,\mathbf{v}^i\mathbf{\Pi}_i \frac{1}{\nabla^2 - M^2}\mathrm{Tr}\,\mathbf{v}^i\mathbf{\Pi}_i \right] |\Phi\rangle. \tag{15}$$

Now we recall that the physical state can be written as [15]

$$|\Phi\rangle = \overline{\psi}(\mathbf{y})\, P \exp\left(\mathrm{i}g \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z^i \mathbf{A}_i(z) \right) \psi(\mathbf{0})\, |0\rangle. \tag{16}$$

The line integral is along a spacelike path on a fixed time slice, P is the path-ordering prescription and $|0\rangle$ is the physical vacuum state. As in [15], we again restrict our attention to the weak coupling limit. From this and the foregoing Hamiltonian discussion, we then get

$$\langle H \rangle_\Phi = \langle H \rangle_0 + V_1 + V_2, \tag{17}$$

where $\langle H \rangle_0 = \langle 0|H|0\rangle$, and the V_1 and V_2 terms are given by

$$V_1 = \frac{1}{2}\langle \Phi | \int \mathrm{d}^3x\, \mathrm{Tr}\,\mathbf{\Pi}^i\mathbf{\Pi}_i\, |\Phi\rangle, \tag{18}$$

[3] In our gauge-invariant, path-dependent formalism, the gauge fixing procedure is equivalent to the choice of a particular path [15], e.g. a spacelike, straight, path $x^i = \xi^i + \lambda(x - \xi)^i$, on a fixed time slice.

and

$$V_2 = -\frac{\beta^2}{4}\langle\Phi| \int \mathrm{d}^3x\, \mathrm{Tr}\, \mathbf{v}^i \mathbf{\Pi}_i \frac{1}{\nabla^2 - M^2} \mathrm{Tr}\, \mathbf{v}^i \mathbf{\Pi}_i |\Phi\rangle. \tag{19}$$

One immediately sees that the V_1 term is identical to the energy for the Yang–Mills theory. Notwithstanding, in order to put our discussion into context it is useful to summarize the relevant aspects of the analysis described previously [15]. Thus, we then get an Abelian part (proportional to C_F) and a non-Abelian part (proportional to the combination $C_F C_A$). As we have noted before, the Abelian part takes the form

$$V^{(g^2)} = \frac{1}{2} g^2\, \mathrm{Tr}(T^a T_a) \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z^i \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z_i\, \delta(\mathbf{z} - \mathbf{z}'), \tag{20}$$

remembering that the integrals over z^i and z_i' are zero except on the contour of integration. Writing the group factor $\mathrm{Tr}\, T^a T_a = C_F$, expression (20) is given by

$$V^{(g^2)}(L) = -\frac{g^2 C_F}{4\pi L}, \tag{21}$$

where $|\mathbf{y}| \equiv L$. Next, the non-Abelian part may be written as

$$V^{(g^4)} = \mathrm{Tr} \int \mathrm{d}^3x \langle 0|\mathbf{I}^i \mathbf{I}_i|0\rangle, \tag{22}$$

where

$$I^{ai} = g^2 f^a_{bc} T^b \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z^k \int_0^1 \mathrm{d}\lambda\, A_k^c(z) z^i \delta(\mathbf{x} - \lambda\mathbf{z}). \tag{23}$$

It should be noted that, by using spherical coordinates, expression (23) reduces to

$$I^{ai} = g^2 f^a_{bc} T^b \frac{\mathbf{z}^i}{|\mathbf{z}|} \frac{1}{|\mathbf{x}|^2} \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z^k A_k^c(z) \sum_{lm} Y^*_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi). \tag{24}$$

Putting this back into equation (22), we obtain

$$V^{g^4}(L) = -C_A C_F \left(\frac{g^4}{2L}\right) \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z^i \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z'^j D_{ij}(z, z'). \tag{25}$$

Here $D_{ij}(z, z')$ stands for the propagator, which is diagonal in colour and taken in an arbitrary gauge. Following our earlier discussion, we choose the Feynman gauge. As a consequence, expression (25) then becomes

$$V^{g^4}(L) = -g^4 \frac{1}{4\pi^2} C_A C_F \frac{1}{L} \log(\Lambda L), \tag{26}$$

where Λ is a cutoff. Then, the V_1 term takes the form

$$V_1 = -g^2 C_F \frac{1}{4\pi L}\left(1 + \frac{g^2}{\pi} C_A \log(\Lambda L)\right). \tag{27}$$

It is important to realize that our calculation was based only on the Hamiltonian and on the geometrical requirement that the fermion–antifermion state be invariant under gauge transformations. From (27) we see that the term of order g^2 is just the Coulomb energy due to the colour charges of the quarks. The correction term of order g^4 represents an increase of the energy due to the vacuum fluctuations of the gauge fields.

The task is now to evaluate the V_2 term, which is given by

$$V_2 = -\frac{\beta^2}{4}\langle\Phi| \int \mathrm{d}^3x\, \mathrm{Tr}\, \mathbf{v}^i \mathbf{\Pi}_i \frac{1}{\nabla^2 - M^2} \mathrm{Tr}\, \mathbf{v}^j \mathbf{\Pi}_j |\Phi\rangle. \tag{28}$$

Once again, from our above Hamiltonian structure we have an Abelian contribution and a non-Abelian contribution; in other words,

$$V_2^{(\mathrm{Ab.})} = -\frac{\beta^2}{4} g^2 \mathrm{Tr}\, \mathbf{v}^i \mathbf{v}_i \int \mathrm{d}^3x \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z'^i \delta(\mathbf{x}-\mathbf{z}') \frac{1}{\nabla_x^2 - M^2} \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z^i \delta(\mathbf{x}-\mathbf{z}), \tag{29}$$

and

$$V_2^{(\mathrm{Non\text{-}Abelian})} = \frac{\beta^2 g^4}{4} \mathrm{Tr}(T^b T^d) f_{pbc} f_{qd}^c v^{pi} v_i^q \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z^l \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z'^k D_{lk}(\mathbf{z}, \mathbf{z}') \times \int_{\mathbf{0}}^{\mathbf{z}'} \mathrm{d}u^j \int_{\mathbf{0}}^{\mathbf{z}} \mathrm{d}v_j G(\mathbf{u}, \mathbf{v}), \tag{30}$$

as before, and $D_{lk}(z, z')$ represents the propagator. Here, G is the Green function

$$G(\mathbf{u}, \mathbf{v}) = \frac{1}{4\pi} \frac{\mathrm{e}^{-M|\mathbf{z}'-\mathbf{z}|}}{|\mathbf{z}'-\mathbf{z}|}. \tag{31}$$

This Green function is, in momentum space,

$$\frac{1}{4\pi} \frac{\mathrm{e}^{-M|\mathbf{u}-\mathbf{v}|}}{|\mathbf{u}-\mathbf{v}|} = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{u}-\mathbf{v})}}{\mathbf{k}^2 + M^2}. \tag{32}$$

By means of equation (32) and remembering that the integrals over z^i and z'^i are zero except on the contour of integration, the term (29) reduces to the linearly increasing potential [5], that is,

$$V_2^{(\mathrm{Ab.})} = \frac{\beta^2 g^2}{16\pi} \mathrm{Tr}(\mathbf{v}^i \mathbf{v}_i) L \log\left(1 + \frac{\Lambda^2}{M^2}\right), \tag{33}$$

We now proceed to calculate the $V_2^{(\mathrm{Non\text{-}Abelian})}$ term. As before, we will use the Green function (32) in momentum space to handle the integral in equation (30). Following our earlier procedure [5], equation (30) is further rewritten as

$$V_2^{(\mathrm{Non\text{-}Abelian})} = \frac{\beta^2 g^4}{8} \mathrm{Tr}(T^b T^d) f_{pbc} f_{qd}^c v^{pi} v_i^q \log\left(1 + \frac{\Lambda^2}{M^2}\right) \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z^l \int_{\mathbf{0}}^{\mathbf{y}} \mathrm{d}z'^k |\mathbf{z}| D_{lk}(\mathbf{z}, \mathbf{z}'). \tag{34}$$

Now, we move on to compute the integral (34). As in the previous calculation, we choose $D_{lk}(\mathbf{z}, \mathbf{z}')$ in the Feynman gauge. Thus the $V_2^{(\mathrm{Non\text{-}Abelian})}$ term is

$$V_2^{(\mathrm{Non\text{-}Abelian})} = \frac{\beta^2 g^4}{8} \mathrm{Tr}(T^b T^d) f^{pbc} f_{qd}^c v^{pi} v_i^q L \log\left(1 + \frac{\Lambda^2}{M^2}\right), \tag{35}$$

after subtracting the self-energy terms.

From (33) and (35) we then get

$$V_2 = \frac{\beta^2 g^2}{8}\left[\frac{1}{2\pi} \mathrm{Tr}(\mathbf{v}^i \mathbf{v}_i) + g^2 \mathrm{Tr}(T^b T^d) f_{pbc} f_{qd}^c v^{pi} v_i^q\right] L \log\left(1 + \frac{\Lambda^2}{M^2}\right). \tag{36}$$

By putting together equations (27) and (36), we obtain for the total interquark potential

$$V = -\frac{g^2 C_F}{4\pi L}\left[1 + \frac{g^2}{\pi} C_A \log(\Lambda L)\right] + \frac{\beta^2 g^2}{8}\left[\frac{1}{2\pi} \mathrm{Tr}(\mathbf{v}^i \mathbf{v}_i) + g^2\, \mathrm{Tr}(T^b T^d) f_{pbc} f_{qd}^c v^{pi} v_i^q\right] L \log\left(1 + \frac{\Lambda^2}{M^2}\right). \tag{37}$$

It must be observed that the rotational symmetry is restored in the resulting form of the potential, although the external fields break the isotropy of the problem in a manifest way.

It should be remarked that this feature is also shared by the corresponding Abelian interaction energy [5]. As we have noted before this improves the situation as compared to the 'spaghetti vacuum' model [6] where rotational symmetry seems to be very difficult to restore.

Now we recall the calculation reported in [2] by taking into account topological nontrivial sectors in $U(1)$ gauge theory

$$V(L) = -\frac{e^2}{4\pi}\frac{1}{L} + \sigma L. \tag{38}$$

We immediately see that the result (37) is exactly the one obtained in [2]. It is interesting to note that even if (6) is an effective model, extending the Abelian one discussed in [5], it is able to reproduce the correct form of the Cornell potential (38). As such it deserves some further investigation.

2.2. Chromo-electric case

Now we focus on the case $\mathbf{v}^{0i} = 0$ and $\mathbf{v}^{ij} \neq 0$ (referred to as the chromo-electric one in what follows). The corresponding Lagrangian density reads

$$\mathcal{L} = -\frac{1}{4}\operatorname{Tr}\mathbf{f}_{\mu\nu}\mathbf{f}^{\mu\nu} + \frac{\beta^2}{16}\operatorname{Tr}\mathbf{v}^{ij}\mathbf{f}_{ij}\frac{1}{\nabla^2 - m_A^2}\operatorname{Tr}\mathbf{v}^{kl}\mathbf{f}_{kl} - \operatorname{Tr}\mathbf{A}_0\mathbf{J}^0, \tag{39}$$

where $\mu, \nu = 0, 1, 2, 3$ and $i, j, k, l = 1, 2, 3$. Here again, the quantization is carried out using Dirac's procedure. We can thus write the canonical momenta $\Pi^{a\mu} = -f^{a0\mu}$, which results in the usual primary constraint $\Pi^{a0} = 0$ and $\Pi^{ai} = f^{ai0}$. Defining the electric and magnetic fields by $E^{ai} = f^{i0}$ and $B^{ak} = -\frac{1}{2}\varepsilon^{kij} f^{aij}$, respectively, the canonical Hamiltonian is thus

$$H_C = \int d^3x \left[\frac{1}{2}\operatorname{Tr}\mathbf{E}^i\mathbf{E}_i + \frac{1}{2}\operatorname{Tr}\mathbf{B}^i\mathbf{B}_i - \frac{\beta^2}{16}\varepsilon_{ijm}\varepsilon_{kln}\operatorname{Tr}\mathbf{v}^{ij}\,\mathbf{B}^m\frac{1}{\nabla^2 - m_A^2}\operatorname{Tr}\mathbf{v}^{kl}\,\mathbf{B}^n - \operatorname{Tr}\mathbf{A}_0(\partial_i\mathbf{\Pi}^i - \mathbf{J}^0)\right]. \tag{40}$$

It is straightforward to see that the constrained structure for the gauge field is identical to the usual Yang–Mills theory. However, in order to put the discussion into the context of this paper, it is convenient to mention the relevant aspects of the analysis described previously [15]. Therefore, we pass now to the calculation of the interaction energy.

As done above, our objective is now to calculate the expectation value of the Hamiltonian in the physical state $|\Phi\rangle$. In other words,

$$\langle H\rangle_\Phi = \langle\Phi|\frac{1}{2}\int d^3x\operatorname{Tr}\mathbf{E}^i\mathbf{E}_i|\Phi\rangle. \tag{41}$$

Taking into account the above Hamiltonian structure, the interaction takes the form

$$\langle H\rangle_\Phi = \langle H\rangle_0 + V_1, \tag{42}$$

where $\langle H\rangle_0 = \langle 0|H|0\rangle$. Accordingly, the potential reads

$$V_1 = -\frac{g^2 C_F}{4\pi L}\left(1 + \frac{g^2}{\pi}C_A\log(\Lambda L)\right). \tag{43}$$

3. Final remarks

In summary, we have considered the confinement versus screening issue for a non-Abelian theory with a coupling $\varepsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F^a_{\alpha\beta}$, in the case when there are nontrivial constant expectation values for the gauge field strength $F^a_{\mu\nu}$. The constant gauge field configuration is a solution of the classical equations of motion.

It was shown that in the case when $\langle F^a_{\mu\nu}\rangle$ is chromo-electric-like no unexpected features are found. Indeed, the resulting static potential remains Coulombic. More interestingly, it was shown that when $\langle F^a_{\mu\nu}\rangle$ is chromo-magnetic-like the potential between static charges displays a Coulomb piece plus a linear confining piece. An analogous situation in the Abelian case may be recalled [5]. Also, a common feature of these models (Abelian and non-Abelian) is that the rotational symmetry is restored in the resulting interaction energy.

We recall that the effective action (7) is the expansion of (6) up to second order in the fluctuation field $\mathbf{f}_{\mu\nu}$; thus one could wonder about the effect of including higher order terms. Even if an explicit calculation is well beyond the purpose of this paper, some general comments can be given. We know that higher order quantum effects renormalize coupling constants by making them scale dependent. It is easy to see that the logarithmic correction to the Coulombic term in (37) is nothing but the first-order expansion of the effective, running, coupling constant

$$g^2_{\mathrm{eff}}(\Lambda L) = \frac{g^2\pi}{1 - \frac{g^2 C_A}{\pi}\log(\Lambda L)}. \tag{44}$$

g^2_{eff} exhibits the expected *asymptotically free* behaviour at short distance characterizing the non-Abelian character of the strong interaction. This result marks a clear difference from the Abelian case, where the electric charge increases at short distance. Similar renormalization effects are expected for the string tension σ, as well. However, the $\log\left(1+\frac{\Lambda^2}{M^2}\right)$ factor in the second term in (37) is L-independent. We are confident that higher order corrections will preserve this feature leading to a still confining static potential of the form

$$V(L) = -\frac{g^2_{\mathrm{eff}}(\Lambda L)C_F}{4L} + \sigma_{\mathrm{eff}}(\Lambda^2/M^2)L. \tag{45}$$

An explicit check of (45) will be addressed in a future work.

We conclude by noting that our result agrees with the monopole plasma mechanism [1, 2]. However, although both approaches lead to confinement, the above analysis reveals that the mechanism of obtaining a linear potential is quite different. As already mentioned, in this work we have exploited the similarity between the tree level mechanism that leads to confinement here and the nonperturbative mechanism (caused by monopoles) which gives confinement in *QCD*.

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